Multiple Choice

1 B) As $\overrightarrow{AB}=\overrightarrow{CD}, \overrightarrow{CQ}=\overrightarrow{DP}, \overrightarrow{AB}+\overrightarrow{CQ}=\overrightarrow{CP}$.

2 A) Let $u=x^5, dv=e^{7x}dx,$ then $du=5x^4dx, v=\frac{e^{7x}}{7}$

$$\int x^5e^{7x}dx=\frac{x^5e^{7x}}{7}-\int 5x^4\frac{e^{7x}}{7}dx.$$

3 B) $\overrightarrow{AB}=\begin{pmatrix}4\\2\\5\end{pmatrix}-\begin{pmatrix}-2\\2\\1\end{pmatrix}=2\begin{pmatrix}3\\0\\2\end{pmatrix}, \therefore \underset{\sim}{r}=\begin{pmatrix}4\\2\\5\end{pmatrix}+\lambda\begin{pmatrix}3\\0\\2\end{pmatrix}.$

4 C) Contrapositive and the statement are equivalent.

5 D) e.g. $-3<2$, but $\frac{1}{-3}\not>\frac{1}{2}$.

6 A) If k is real, then $2-i$ is also a root.
In (A), $\sum\alpha=(2+i)+(2-i)+\alpha=4, \therefore \alpha=0,$
which satisfies the equation in (A).

7 D) Due to $t\underset{\sim}{k}$, it circulates about the z-axis, $\therefore$ Only (C) and (D) match.
When $t=0, \underset{\sim}{r}=-5\underset{\sim}{i}$, i.e. -5 on the x-axis.

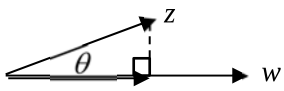
When $t=\frac{\pi}{2}, \underset{\sim}{r}=5\underset{\sim}{j}+\frac{\pi}{2}\underset{\sim}{k}, \therefore$ (D) is correct.

8 D) The acceleration must be centripetal, $\therefore$ Only (C) and (D). As its velocity increases, (D) is correct.

9 B) Taking card 4 is obvious. Using contrapositive, 'if it does not have WIN, the card is not RED', card 3 is taken.

10 C) The projection of $\underset{\sim}{z}$ onto $\underset{\sim}{w}$ has length $|z|\cos\theta$,

where $\cos\theta=\frac{\underset{\sim}{z}.\underset{\sim}{w}}{|z||w|}$. Thus, its length $=|z|\frac{\underset{\sim}{z}.\underset{\sim}{w}}{|z||w|}=\frac{\underset{\sim}{z}.\underset{\sim}{w}}{|w|}$.



$\therefore$ The projection vector of $\underset{\sim}{z}$ onto $\underset{\sim}{w}$ is $\frac{\underset{\sim}{z}.\underset{\sim}{w}}{|w|}\frac{\underset{\sim}{w}}{|w|}=\frac{\underset{\sim}{z}.\underset{\sim}{w}}{|w|^2}\underset{\sim}{w}$.

But $\frac{\underset{\sim}{z}.\underset{\sim}{w}}{|w|^2}=\frac{\text{Re}(z\overline{w})}{|w|^2}=\text{Re}\left(\frac{z}{w}\right)$, in complex numbers.

$\therefore$ The projection of z onto $w=\text{Re}\left(\frac{z}{w}\right)w$.

Question 11

(a) $zw=12e^{i\left(\frac{\pi}{2}+\frac{\pi}{6}\right)}=12e^{i\frac{2\pi}{3}}$.

(b) $\sum_{n=1}^{5}i^n=i+i^2+i^3+i^4+i^5=i-1-i+1+i=i.$

(c) $\cos\theta=\frac{\underset{\sim}{a}.\underset{\sim}{b}}{|a||b|}=\frac{-6+0+8}{\sqrt{4+16}\sqrt{9+1+4}}=\frac{2}{\sqrt{280}}=\frac{1}{\sqrt{70}}$,
$\therefore \theta=83.1°$.

(d) $\sqrt{-i}=\frac{1}{\sqrt{2}}\sqrt{1+i^2-2i}=\frac{\sqrt{(1-i)^2}}{\sqrt{2}}=\pm\left(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)$.

For $z^2+2z+1=i=0$.

$$z=\frac{-2\pm\sqrt{4-4(1+i)}}{2}=-1\pm\sqrt{-i}=-1\pm\frac{1-i}{\sqrt{2}}$$

$$=\frac{-\sqrt{2}+1-i}{\sqrt{2}} \text{ or } \frac{-\sqrt{2}-1+i}{\sqrt{2}}.$$

(e) $\frac{\overline{z}}{w}=\frac{5-i}{2-4i}=\frac{5-i}{2-4i}\times\frac{2+4i}{2+4i}=\frac{10+4+20i-2i}{4+16}$
$=\frac{14+18i}{20}=\frac{7+9i}{10}$.

(f) $\frac{3x^2-5}{(x-2)(x^2+x+1)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+x+1}$, where

$A=\lim_{x\to 2}\frac{3x^2-5}{x^2+x+1}=\frac{7}{7}=1.$

$B=2$, by comparing the coefficients of x^3,
$C=3$, by comparing the constants.

$$\therefore \frac{3x^2-5}{(x-2)(x^2+x+1)}=\frac{1}{x-2}+\frac{2x+3}{x^2+x+1}.$$

Question 12

(a) $\int \frac{2x+3}{x^2+2x+2}dx = \int \frac{2x+2}{x^2+2x+2}dx + \int \frac{1}{(x+1)^2+1}dx$

$= \ln\left|x^2+2x+2\right| + \tan^{-1}(x+1) + C.$

(b) (i) The converse of 'If n^2 is even then n is even' is 'If n is even then n^2 is even'.

(ii) Let $n = 2k, k \in R$, then $n^2 = 4k^2$, which is even.

(c) The 2 lines intersect when $\underset{\sim}{r_1} = \underset{\sim}{r_2}$ for some values of λ or μ,

$$\begin{pmatrix}-2\\1\\3\end{pmatrix} + \lambda\begin{pmatrix}1\\0\\2\end{pmatrix} = \begin{pmatrix}4\\-2\\q\end{pmatrix} + \mu\begin{pmatrix}p\\3\\-1\end{pmatrix}$$

$$\therefore \begin{cases} -2+\lambda = 4+p\mu & (1)\\ 1 = -2+3\mu & (2)\\ 3+2\lambda = q-\mu & (3)\end{cases}$$

Solving (2) gives $\mu = 1$.

The 2 lines are perpendicular when the dot product of their direction vectors equals to 0.

$1\times p + 0\times 3 + 2\times(-1) = 0, \therefore p = 2.$

Substituting $p = 2, \mu = 1$ to (1) gives $\lambda = 8$.

$\therefore$ (3) gives $19 = q - 1, \therefore q = 20$.

(d) Let $n = 9$, $\text{LHS} = \sqrt{9!} \simeq 602.4$, $\text{RHS} = 2^9 = 512$.

Since $602.4 > 512$, the statement is true for $n = 9$.

Assume $\sqrt{n!} > 2^n$ for some value of $n \in J$.

RTP $\sqrt{(n+1)!} > 2^{n+1}$.

$\text{LHS} = \sqrt{n!}\sqrt{n+1}$

$> 2^n\sqrt{n+1}$

$> 2^n \times 2$, since $\sqrt{n+1} > 2$, for $n > 9$

$= 2^{n+1} = \text{RHS}$.

$\therefore \sqrt{n!} > 2^n$ for all integers $n \geq 9$.

(e) (i) $\overrightarrow{HA} = -\overrightarrow{HC}, \overrightarrow{HB} = -\overrightarrow{HD}$

$\therefore \overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} + \overrightarrow{HD} = 0.$

(ii) Let G be a point on HS.

$\overrightarrow{GA} = \overrightarrow{GH} + \overrightarrow{HA}, \overrightarrow{GB} = \overrightarrow{GH} + \overrightarrow{HB}, \overrightarrow{GC} = \overrightarrow{GH} + \overrightarrow{HC}$

and $\overrightarrow{GD} = \overrightarrow{GH} + \overrightarrow{HD}$.

$\therefore \overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} + \overrightarrow{GD} + \overrightarrow{GS}$

$= \overrightarrow{GH} + \overrightarrow{HA} + \overrightarrow{GH} + \overrightarrow{HB} + \overrightarrow{GH} + \overrightarrow{HC} + \overrightarrow{GH} + \overrightarrow{HD} + \overrightarrow{GS}$

$= 4\overrightarrow{GH} + \overrightarrow{GS}$.

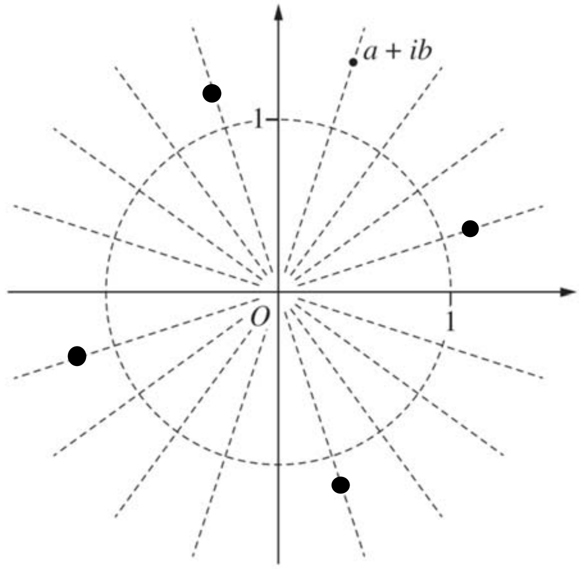
(iii) From (ii), $4\overrightarrow{HG} = \overrightarrow{GS}, \therefore \overrightarrow{HG} = \frac{1}{5}\overrightarrow{HS}$.

$\therefore \lambda = \frac{1}{5}$.

Question 13

(a) Noting that $|a+ib| > 1, 1 < \left|\sqrt[4]{a+ib}\right| < |a+ib|$, and

$\arg\left(\sqrt[4]{a+ib}\right) = \frac{\arg(a+ib)+k2\pi}{4}, k = 0,1,2,3.$



(b) Let $u^2 = x^2 - 9, 2udu = 2xdx$.

When $x = \sqrt{10}, u = 1$. When $x = \sqrt{13}, u = 2$.

$\int_{\sqrt{10}}^{\sqrt{13}} x^3\sqrt{x^2-9}dx = \int_1^2 (u^2+9)u^2du$

$= \int_1^2 (u^4+9u^2)du$

$= \left[\frac{u^5}{5} + 3u^3\right]_1^2$

$= \left(\frac{32}{5}+24\right) - \left(\frac{1}{5}+3\right)$

$= \frac{136}{5}$.

(c) (i) By IBP, let $u = (\ln x)^n, du = n(\ln x)^{n-1}\frac{1}{x}dx$.

Let $dv = dx, v = x$.

$I_n = \int_1^e (\ln x)^n dx$

$= \left[x(\ln x)^n\right]_1^e - n\int_1^e (\ln x)^{n-1}dx$

$= e - nI_{n-1}$.

(ii) V_A = Volume of cylinder of radius 1, and height 1 $-\frac{1}{2}$ volume of sphere of radius 1

$=\pi-\frac{1}{2}\times\frac{4}{3}\pi$

$=\frac{\pi}{3}\text{u}^3.$

V_B = Volume of cylinder of radius 1, and

height $(e-1)-\pi\int_1^e(\ln x)^2dx$

$=\pi(e-1)-\pi I_2.$

From part (i),

$I_2=e-2I_1$

$=e-2(e-I_0)$

$=-e+2I_0$

$=-e+2(e-1),\text{since } I_0=\int_1^e dx=e-1,$

$=e-2.$

$\therefore V_B=\pi(e-1)-\pi(e-2)$

$=\pi.$

$\therefore V_A:V_B=\frac{\pi}{3}:\pi$

$=1:3.$

(d) (i) $\ddot{x}=\frac{vdv}{dx}=-4(x-3).$

$\int_{-8}^{v}vdv=\int_0^x-4(x-3)dx$ (taking $v=-8$, as it moves towards the origin from $x=5.5$ m)

$\left[\frac{v^2}{2}\right]_{-8}^{v}=-4\left[\frac{x^2}{2}-3x\right]_0^x$

$v^2-64=-8\left(\frac{x^2}{2}-3x\right)$

$v^2=-4x^2+24x+64$

$=4\left(16+6x-x^2\right)$

$=4(8-x)(x+2).$

The particle oscillates between 2 values of x when $v=0$, i.e. when $x=-2$ and $x=8$.

(ii) $v=\frac{dx}{dt}=-2\sqrt{16+6x-x^2}$, taking negative sign as it moves towards the origin from $x=5.5$ m.

$\int_{5.5}^{0}\frac{-dx}{\sqrt{16+6x-x^2}}=2\int_0^T dt$

$2T=\int_{5.5}^{0}\left(\frac{-1}{\sqrt{25-(x-3)^2}}\right)dx$

$=\left[\cos^{-1}\frac{x-3}{5}\right]_{5.5}^{0}$

$=\cos^{-1}\left(-\frac{3}{5}\right)-\cos^{-1}\left(\frac{1.5}{5}\right)$

$=0.948.$

$\therefore T=0.47$ sec.

Question 14

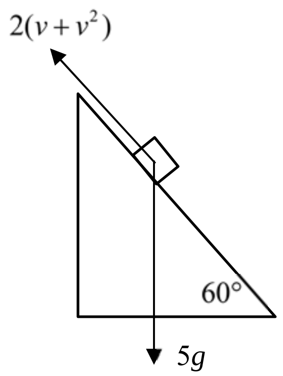
(a) Let $t = \tan\frac{x}{2}, \cos x = \frac{1-t^2}{1+t^2}$.

$dt = \frac{1}{2}\sec^2\frac{x}{2}dx, \therefore dx = \frac{2dt}{\sec^2\frac{x}{2}} = \frac{2dt}{1+t^2}$.

When $x = 0, t = 0$. When $x = \frac{\pi}{2}, t = 1$.

$$\int_0^{\frac{\pi}{2}} \frac{1}{3+5\cos x}dx = \int_0^1 \frac{1}{3+\frac{5(1-t^2)}{1+t^2}}\frac{2dt}{1+t^2}$$
$$= \int_0^1 \frac{2}{8-2t^2}dt$$
$$= \int_0^1 \frac{1}{(2+t)(2-t)}dt$$
$$= \frac{1}{4}\int_0^1\left(\frac{1}{2+t}+\frac{1}{2-t}\right)dt$$
$$= \frac{1}{4}\left[\ln\frac{2+t}{2-t}\right]_0^1$$
$$= \frac{1}{4}\ln 3.$$

(b)



(i) The component of the gravity down the slope is

$5g\sin 60° = \frac{5g\sqrt{3}}{2}$.

$\therefore$ The resultant force down the slope is

$5a = \frac{5g\sqrt{3}}{2} - 2v - 2v^2$.

(ii) The object will slide down the slope with constant speed when $a = 0$.

$\therefore 2v^2 + 2v - 25\sqrt{3} = 0$, taking $g = 10$.

$v = \frac{-1\pm\sqrt{1+50\sqrt{3}}}{2} = -5.2$ or $+4.2$.

Taking $v = 4.2$ ms^{-1}, as going down the slope has positive sign.

(c) (i) $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$, by De Moivre's theorem.

By expanding, using $c = \cos\theta, s = \sin\theta$

$(c+is)^5 = c^5 + i5c^4s - 10c^3s^2 - 10ic^2s^3 + 5cs^4 + is^5$.

By equating the real parts,

$$\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$$
$$= c^5 - 10c^3(1-c^2) + 5c(1-c^4)^2$$
$$= c^5 - 10c^3 + 10c^5 + 5c(1-2c^2+c^4)$$
$$= c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$$
$$= 16c^5 - 20c^3 + 5c$$
$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(ii) $e^{\frac{i\pi}{10}} = \cos\frac{\pi}{10} + i\sin\frac{\pi}{10}$.

$\text{Re}\left(e^{\frac{i\pi}{10}}\right) = \cos\frac{\pi}{10}$.

From part (i), let $\theta = \frac{\pi}{10}$,

$16\cos^5\frac{\pi}{10} - 20\cos^3\frac{\pi}{10} + 5\cos\frac{\pi}{10} = \cos\frac{5\pi}{10} = 0$.

$\therefore 16\cos^4\frac{\pi}{10} - 20\cos^2\frac{\pi}{10} + 5 = 0$, as $\cos\frac{\pi}{10} \neq 0$.

$\therefore \cos^2\frac{\pi}{10} = \frac{20\pm\sqrt{400-320}}{32} = \frac{5\pm\sqrt{5}}{8}$.

Taking $\cos^2\frac{\pi}{10} = \frac{5+\sqrt{5}}{8}$, since $\cos^2\frac{\pi}{10} > 0$.

$\therefore \cos\frac{\pi}{10} = \sqrt{\frac{5+\sqrt{5}}{8}}$, since $\cos\frac{\pi}{10} > 0$ as $\frac{\pi}{10}$ lies in the first quadrant.

$\therefore \text{Re}\left(e^{\frac{i\pi}{10}}\right) = \sqrt{\frac{5+\sqrt{5}}{8}}$.

Question 15

(a) (i) $\sqrt{abc} \le \dfrac{ab+c}{2}$

But $\sqrt{a^2b^2} = ab \le \dfrac{a^2+b^2}{2}$.

$\therefore \sqrt{abc} \le \dfrac{\frac{a^2+b^2}{2}+c}{2}$

$= \dfrac{a^2+b^2+2c}{4}$.

(ii) From (i), $\sqrt{abc} \le \dfrac{a^2+b^2+2c}{4}$.

Similarly, $\sqrt{abc} \le \dfrac{a^2+c^2+2b}{4}$,

and $\sqrt{abc} \le \dfrac{c^2+b^2+2a}{4}$.

Adding the 3 lines above gives

$3\sqrt{abc} \le \dfrac{2a^2+2b^2+2c^2+2a+2b+2c}{4}$.

$\therefore \sqrt{abc} \le \dfrac{a^2+b^2+c^2+a+b+c}{6}$.

(b) (i) Let $n = 2k-1, k \in J, k \ge 1$.

$t_n = \dfrac{(2k-1)(2k)}{2} = 2k^2 - k$.

$\therefore t_1, t_3, t_5$ and so on are hexagonal numbers.

(ii) Let $n = 2k, k \in J, k \ge 1$.

$t_n = \dfrac{2k(2k+1)}{2} = 2k^2 + k$.

$\therefore t_2, t_4, t_6$ and so on are not hexagonal numbers.

(c) (i) Let the direction upwards be positive, the force experienced by the object is

$ma = -mg - kv^2$.

$\therefore a = -\left(g+kv^2\right)$, since $m = 1$ kg.

$a = \dfrac{dv}{dt} = -\left(g+kv^2\right)$.

$\int dt = \int \dfrac{-dv}{g+kv^2}$

$t = \dfrac{-1}{\sqrt{gk}}\tan^{-1} v\sqrt{\dfrac{k}{g}} + C$.

When $t = 0, v = u, \therefore C = \dfrac{1}{\sqrt{gk}}\tan^{-1} u\sqrt{\dfrac{k}{g}}$

$\therefore t = \dfrac{-1}{\sqrt{gk}}\tan^{-1} v\sqrt{\dfrac{k}{g}} + \dfrac{1}{\sqrt{gk}}\tan^{-1} u\sqrt{\dfrac{k}{g}}$.

Maximum height is reached when $v = 0$,

$\therefore t = \dfrac{1}{\sqrt{gk}}\tan^{-1} u\sqrt{\dfrac{k}{g}}$.

(ii) $a = \dfrac{vdv}{dx} = -\left(g+kv^2\right)$.

$\int dx = \int \dfrac{-vdv}{g+kv^2}$

$x = \dfrac{-1}{2k}\ln\left(g+kv^2\right) + C$.

When $x = 0, v = u, \therefore C = \dfrac{1}{2k}\ln\left(g+ku^2\right)$

$\therefore x = \dfrac{1}{2k}\ln\dfrac{g+ku^2}{g+kv^2}$.

Maximum height is reached when $v = 0$,

$\therefore H = \dfrac{1}{2k}\ln\dfrac{g+ku^2}{g}$.

(d) $5^n = (2+3)^n$

$= 2^n + n\times 2^{n-1}\times 3 + \ldots + n\times 2\times 3^{n-1} + 3^n$.

$> 2^n + 3^n$, for all $n \ge 2$.

$\therefore 5^n \ne 2^n + 3^n$, for all $n \ge 2$.

Question 16

(a) (i) By triangular inequality,

$$\left|x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}\right| \le \left|x\underset{\sim}{i}\right|+\left|y\underset{\sim}{j}\right|+\left|z\underset{\sim}{k}\right| = |x|+|y|+|k|.$$

But P lies on the unit sphere, $\left|\overline{OP}\right| = \left|x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}\right| = 1$.

$\therefore |x|+|y|+|k| \ge 1$.

(ii) $\underset{\sim}{a}.\underset{\sim}{b} = a_1b_1+a_2b_2+a_3b_3$

$= |a||b|\cos\theta$, where θ is the angle between the 2 vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

$\therefore |a_1b_1+a_2b_2+a_3b_3| \le |a||b|$

$= \sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}$.

(iii) Let $\underset{\sim}{a} = \begin{pmatrix}1\\1\\1\end{pmatrix}, \underset{\sim}{b} = \begin{pmatrix}|x|\\|y|\\|z|\end{pmatrix}$, where $P(x,y,z)$ lies in the unit sphere, i.e. $\sqrt{x^2+y^2+z^2} = 1$.

From part (ii),

$$\big|1\times|x|+1\times|y|+1\times|z|\big| \le \sqrt{1^2+1^2+1^2}\sqrt{x^2+y^2+z^2}.$$

But $\big|1\times|x|+1\times|y|+1\times|z|\big| = |x|+|y|+|z|$.

$\therefore |x|+|y|+|z| \le \sqrt{3}$.

(b) $\underset{\sim}{v}(t) = \begin{cases} u\cos\theta \\ -gt+u\sin\theta \end{cases}$

We want $\dfrac{-\frac{1}{2}gt^2+ut\sin\theta}{ut\cos\theta} \times \dfrac{-gt+u\sin\theta}{u\cos\theta} = -1$.

$$\frac{-\frac{1}{2}gt+u\sin\theta}{u\cos\theta} \times \frac{-gt+u\sin\theta}{u\cos\theta} = -1$$

$$\frac{1}{2}g^2t^2 - \frac{3}{2}ugt\sin\theta + u^2\sin^2\theta = -u^2\cos^2\theta.$$

$$\frac{1}{2}g^2t^2 - \frac{3}{2}ugt\sin\theta = -u^2, \text{ since } \cos^2\theta+\sin^2\theta = 1.$$

$$g^2t^2 - 3ugt\sin\theta + 2u^2 = 0.$$

Solving the quadratic above,

$$t = \frac{3ug\sin\theta \pm \sqrt{9u^2g^2\sin^2\theta - 8g^2u^2}}{2g^2}$$

$$= \frac{3u\sin\theta}{2g} \pm \frac{u\sqrt{9\sin^2\theta-8}}{2g}$$

$$= \frac{u}{2g}\left(3\sin\theta \pm \sqrt{9\sin^2\theta-8}\right) \qquad (*)$$

Noting that $\sqrt{9\sin^2\theta-8} < 3\sin\theta$, both times t are positive.

Since the time of flight occurs when $-\dfrac{gT^2}{2}+uT\sin\theta = 0$,

i.e. $T = \dfrac{2u\sin\theta}{g}$, to prove that both times occur during the time of flight, we need to prove that the larger of the above two times is smaller than the time of flight. Indeed,

$$t = \frac{u}{2g}\left(3\sin\theta+\sqrt{9\sin^2\theta-8}\right)$$

$$< \frac{u}{2g}\left(3\sin\theta+\sqrt{9\sin^2\theta-8\sin^2\theta}\right)$$

$$< \frac{2u\sin\theta}{g}.$$

Note: From (*), the equation has 2 real roots when $8 < 9\sin^2\theta < 9$.

$\therefore \dfrac{2\sqrt{2}}{3} < \sin\theta < 1$.

$\therefore 70.5° < \theta < 90°$.

(c) Let $z = x+iy$.

If $0 < \text{Arg}(z) < \dfrac{\pi}{2}, \text{Re}(z) \ge \text{Arg}(z) \Leftrightarrow x \ge \tan^{-1}\dfrac{y}{x}$.

$\tan x \ge \dfrac{y}{x}, \therefore y \le x\tan x, x \ne 0$.

But as RHS $< \dfrac{\pi}{2}, \text{Re}(z) > \text{Arg}(z)$ is true for $x \ge \dfrac{\pi}{2}$.

If $\dfrac{\pi}{2} < \text{Arg}(z) < \pi$. i.e. $\text{Arg}(z) > 0, \text{Re}(z) = x < 0$.

$\therefore$ no solution, i.e. the x-axis, $x<0$, is also not accepted.

If $-\pi < \text{Arg}(z) < -\dfrac{\pi}{2}, \text{Arg}(z) = -\pi+\tan^{-1}\dfrac{y}{x}, \text{Re}(z) = x < 0$.

$\text{Re}(z) \ge \text{Arg}(z) \Leftrightarrow x \ge -\pi+\tan^{-1}\dfrac{y}{x}$.

$x+\pi \ge \tan^{-1}\dfrac{y}{x}, \therefore y \ge x\tan(x+\pi)$.

Since the period of $\tan x$ is π, the graph of $y = x\tan x$ and $y = x\tan(x\pm\pi)$ are the same.

If $-\dfrac{\pi}{2} < \text{Arg}(z) < 0, \text{Re}(z) = x > 0$ i.e. $\text{Re}(z) > \text{Arg}(z)$ for all real x.